

Research Article

Single and Interval-Valued Hybrid Enthalpy Fuzzy Sets and a TOPSIS Approach for Multicriteria Group Decision Making

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The concept of entropy is one of the most important notions of the information theory. Entropy quantifies the amount of uncertainty involved in the value of a random variable or the outcome of a random process. Shannon's entropy is one of the most useful entropy types. The notion of enthalpy is the information energy expressed by the complement of Shannon's entropy. In this paper, we propose the concept of interval-valued hybrid enthalpy fuzzy set by modifying single and interval-valued fuzzy multisets. In this context, an interval-valued hybrid enthalpy fuzzy set contains information about both the data and their entropy. We also provide a cosine similarity measure between interval-valued hybrid enthalpy fuzzy sets. Using this cosine similarity measure, we propose a TOPSIS approach for multicriteria group decision making. Moreover, we apply the proposed TOPSIS method to a research assistant selection problem, and we compare the result with the result of a classical TOPSIS method.

1. Introduction

The primary principle behind the notion of fuzzy set is that an element's membership degree cannot always be 0 or 1 to a set, but they may instead be between 0 and 1. To model this pneumonia, Zadeh [1] generalized the concept of characteristic function of crisp sets to the concept of membership function of fuzzy sets. As a result, ambiguous information that cannot be expressed without loss of information can be easily modeled via fuzzy sets. Later, Zadeh [2] combined fuzzy set theory with interval mathematics to better express the uncertain information and proposed the notion of interval-valued fuzzy set (IVFS). Actually, interval mathematics is a type of information theory that is linked to fuzzy logic but independent from it. An IVFS is defined with the help of an interval-valued membership function. The main characteristic of an IVFS is that the values of the membership function are intervals rather than exact numbers. The uncertainty reduces in fuzzy environment, when the degrees of membership function is expressed with ranges

instead of certain numbers [3–5]. Moreover, these concepts have been used to solve various multicriteria decision making (MCDM) and multicriteria group decision making (MCGDM) problems by several researchers [6–8]. In a MCDM or MCGDM problem in the fuzzy environment, decision makers (DMs) assess each alternative in terms of conflicting criteria by using the concept of membership function. However, a DM may face some difficulties while determining membership degrees of elements since an exact value has to be assigned to them between 0 and 1.

Yager [9] proposed the concept of fuzzy multiset (FMS) as a generalization of fuzzy sets (FSs). In an FMS, the membership degree of an element is represented as a finite sequence of the same or different fuzzy values and so it prevents the loss of the repetitive information. Later, Pramanik et al. [10] introduced the concept of interval-valued fuzzy multiset (IVFMS). In this fuzzy environment, the membership degree of an element is represented as a finite sequence of closed subintervals of $[0, 1]$. Recently, researchers have focused on hybrid information. For example,

Jun et al. [11] have proposed the concept of cubic sets (CSs) by uniting FSs and IVFSs. Ye et al. [12] have proposed a new fuzzy set concept by integrating single-valued and interval-valued neutrosophic sets in the meaning of fuzzy multiset theory. Regarding a special case of the single and interval-valued hybrid neutrosophic multivalued set (SIVHNMVS) [12], we can introduce the notion of single and interval-valued hybrid fuzzy multiset (SIVHFMS) by considering the truth membership function in SIVHNMVS. The purpose of the present study is to introduce a new hybrid fuzzy set which is called single and interval-valued hybrid enthalpy fuzzy set (SIVHEFS) with the help of single and interval-valued hybrid fuzzy multisets (SIVHFMSs).

An enthalpy value is expressed by the complement of Shannon's entropy [13] which is a useful measurement method, and it is used to measure the uncertainty in randomly distributed data in information analysis. Fu et al. [14] have proposed the concept of entropy fuzzy set for FMSs, and the concept of enthalpy value has been introduced by Ye et al. [15] for neutrosophic multivalued sets. In this paper, we focus on SIVHEFS and its MCGDM applications. A SIVHEFS is characterized with a pair of membership terms. The first complement of this pair is again a pair which consists of the average of the single part of the sequence and the average of the sequence of intervals. The second complement of this pair is also a pair which consists of the enthalpy of the single part of the sequence and the enthalpy of the sequence of intervals. Therefore, a SIVHEFS contains not only the level of the average of the data but also the degree of the uncertainty of the data via enthalpy. Consequently, a SIVHEFS can better model uncertain information by using hybrid data. Our main aim is to introduce a new technique for order of preference by similarity to ideal solution (TOPSIS) method. In this method, it is aimed to find an alternative which is the farthest distance from the ideal worst solution and the shortest distance from the ideal best solution. This method frequently has been used to solve MCDM and MCGDM problems in several fuzzy environments. For example, Ashtiani et al. [16] proposed an interval-valued fuzzy TOPSIS method which was presented for solving MCDM problems in which the weights of criteria are unequal. Gündogdu and Kahraman [17] introduced an interval-valued spherical fuzzy TOPSIS method to solve a MCDM problem. Garg et al. [18] proposed a new TOPSIS method based on the complex interval-valued q -rung orthopair fuzzy set. Wang et al. [19] have introduced a TOPSIS method for interval-valued q -rung dual hesitant fuzzy sets. Huang et al. [20] have presented aggregation operators for spherical fuzzy rough sets (SFR), and they have used them to propose a new TOPSIS algorithm in spherical fuzzy rough environment.

In this paper, our aim is to present a new hybrid fuzzy set which is called SIVHEFS and an enthalpy-TOPSIS method based on the Choquet integral [21]. For this purpose, first, we construct a cosine similarity measure with the help of the Choquet integral to determine the distances used in TOPSIS approach. A similarity measure is an important tool to measure the degree of similarity between two mathematical object, and it has been studied

by many researchers to solve MCDM problems in several fuzzy environment [22–24]. The concept of cosine similarity measure for fuzzy sets is defined with the help of cosine of the angle between the vector representations of fuzzy sets [25]. We also give a score function to compare SIVHEFSs. The reason we use Choquet integral when constructing a similarity measure is that Choquet integral takes into account the interaction between criteria with the help of a fuzzy measure [26]. Therefore, we determine a λ -fuzzy measure [26, 27] and so we can calculate the furthest and shortest distances with the help of the similarity measure. Finally, we apply the proposed TOPSIS method to a research assistant selection problem to demonstrate its feasibility and effectiveness.

The objective of this paper is to propose the concept of SIVHEFS in order to better model the single-valued and interval-valued information given in fuzzy multi environment by using the concepts of enthalpy and average value, and to present a TOPSIS application of this concept with a help of Choquet integral. Main contributions of this paper are summarized as follows:

- (i) We propose a new hybrid fuzzy set which is called SIVHEFS by transforming the notion of SIVHFMS. A SIVHEFS that is based on the average values and the enthalpy value can give reasonable hybrid information about sequences in a FMS and IVFMS.
- (ii) The concept of SIVHEFS reduces the dependence of information on the length of the sequence in FMS and IFMS and presents the hybrid information in a more compact form.
- (iii) The concept of SIVHEFS contains statistical information with the average of the membership sequences as well as enthalpy value which is constructed with the help of Shannon's entropy that calculates the amount of the uncertainty of the information of an event.
- (iv) Choquet integral is a generalization of the arithmetic and weighted means, and it takes into account the interaction between criteria via fuzzy measures. Therefore, the proposed cosine similarity measure is relatively sensitive.
- (v) The proposed score function provides a useful ranking method for intervals thanks to aggregation operators.
- (vi) The developed enthalpy-TOPSIS approach not only improves the decision-making reliability but also supplies a new influential way for DMs.
- (vii) Table 1 gives the comparison of SIVHEFSs with some existing fuzzy sets.

The rest of the paper is organized as follows. In Section 2, we recall the concepts of SIVHFMS and Shannon's entropy and we propose the notion of SIVHEFS. We also recall definitions of the fuzzy measure and Choquet integral. Then, we present a similarity measure based on the Choquet integral between SIVHEFS. In Section 3, we provide a new TOPSIS method and a score function for SIVHEFS. Later,

TABLE 1: Comparison of SIVHEFS with some existing fuzzy sets.

Sets	Single-valued membership sequence	interval-valued membership sequence	Hybrid information	Statistical information	Amount of uncertainty information
FS [1]	×	×	×	×	×
IVFS [2]	×	×	×	×	×
FMS [9]	✓	×	×	×	×
IVFMS [10]	×	✓	×	×	×
SIVHFMS	✓	✓	✓	×	×
SIVHEFS	✓	✓	✓	✓	✓

we give the decision steps and apply it to a research assistant selection problem. We also give a comparative analysis with the classical TOPSIS method. In the last section, we conclude the paper.

2. Preliminaries

In this section, we introduce some fundamental concepts of SIVHFMS and enthalpy. We also define the notion of SIVHEFS, and we provide a cosine similarity measure between SIVHEFSs based on the Choquet integral.

Definition 1. Let $X = \{x_1, \dots, x_n\}$ be a finite set. A single and interval-valued hybrid fuzzy multiset (SIVHFMS) on X is given with

$$A = \left\{ \left\langle x_i, \{\mu_A^j(x_i)\}_{j=1}^{\delta_i+\tau_i} \right\rangle : i = 1, \dots, n \right\}, \quad (1)$$

where $\mu_A^j(x_i) \in [0, 1]$ for any $j = 1, \dots, \delta_i$ and $\mu_A^j(x_i) \in I[0, 1]$ are intervals for any $j = \delta_i + 1, \dots, \delta_i + \tau_i$, i.e.,

$$A = \left\{ \left\langle x_i, \left(\mu_A^1(x_i), \dots, \mu_A^{\delta_i}(x_i), \left[\mu_{AL}^{\delta_i+1}(x_i), \mu_{AU}^{\delta_i+1}(x_i) \right], \dots, \left[\mu_{AL}^{\delta_i+\tau_i}(x_i), \mu_{AU}^{\delta_i+\tau_i}(x_i) \right] \right) \right\rangle : i = 1, \dots, n \right\}. \quad (2)$$

Also, the basic element $\left\langle x_i, \{\mu_A^j(x_i)\}_{j=1}^{\delta_i+\tau_i} \right\rangle$ for a fixed $i = 1, \dots, n$ is given with

$$\alpha = \left\langle \left(\mu_{\alpha}^1, \dots, \mu_{\alpha}^{\delta_i}, \left[\mu_{L_{\alpha}}^{\delta_i+1}, \mu_{U_{\alpha}}^{\delta_i+1} \right], \dots, \left[\mu_{L_{\alpha}}^{\delta_i+\tau_i}, \mu_{U_{\alpha}}^{\delta_i+\tau_i} \right] \right) \right\rangle, \quad (3)$$

which is called a single and interval-valued hybrid fuzzy multivalued (SIVHFMS).

Definition 2. Let $P = \{r_1, \dots, r_n\}$ be a probability distribution on a universal set; then, Shannon's entropy of P is defined by

$$H(P) = - \sum_{i=1}^n r_i \ln(r_i). \quad (4)$$

Let $X = \{x_1, \dots, x_n\}$ be a finite set, and let A be a SIVHFMS on X . To calculate Shannon's entropy of the sequences, the values should be normalized so that their sum is equal to 1. For this purpose, let us define

$$\begin{aligned} SM_A &= \sum_{j=1}^{\delta_i} \mu_A^j(x_i), \\ SM_{AL} &= \sum_{j=\delta_i+1}^{\delta_i+\tau_i} \mu_{AL}^j(x_i), \\ SM_{AU} &= \sum_{j=\delta_i+1}^{\delta_i+\tau_i} \mu_{AU}^j(x_i). \end{aligned} \quad (5)$$

Now, it is clear that

$$\begin{aligned} \sum_{j=1}^{\delta_i} \frac{\mu_A^j(x_i)}{SM_A} &= 1, \\ \sum_{j=\delta_i+1}^{\delta_i+\tau_i} \frac{\mu_{AL}^j(x_i)}{SM_{AL}} &= 1, \\ \sum_{j=\delta_i+1}^{\delta_i+\tau_i} \frac{\mu_{AU}^j(x_i)}{SM_{AU}} &= 1, \end{aligned} \quad (6)$$

for $i = 1, 2, \dots, n$. Now, the values

$$\begin{aligned} C\left(\{\mu_A^j(x_i)\}_{j=1}^{\delta_i}\right) &= 1 + \frac{1}{\delta_i} \frac{1}{SM_A} \sum_{j=1}^{\delta_i} \mu_A^j(x_i) \ln\left(\frac{\mu_A^j(x_i)}{SM_A}\right), \\ C\left(\{\mu_{AL}^j(x_i)\}_{j=\delta_i+1}^{\delta_i+\tau_i}\right) &= 1 + \frac{1}{\tau_i} \frac{1}{SM_{AL}} \sum_{j=\delta_i+1}^{\delta_i+\tau_i} \mu_{AL}^j(x_i) \ln\left(\frac{\mu_{AL}^j(x_i)}{SM_{AL}}\right), \\ C\left(\{\mu_{AU}^j(x_i)\}_{j=\delta_i+1}^{\delta_i+\tau_i}\right) &= 1 + \frac{1}{\tau_i} \frac{1}{SM_{AU}} \sum_{j=\delta_i+1}^{\delta_i+\tau_i} \mu_{AU}^j(x_i) \ln\left(\frac{\mu_{AU}^j(x_i)}{SM_{AU}}\right), \end{aligned} \quad (7)$$

are the enthalpy of (normalized) $\mu_A^j(x_i)$, $\mu_{AL}^j(x_i)$, $\mu_{AU}^j(x_i)$, where C is the enthalpy operator defined by Ye et al. (see [15]).

Definition 3. Let $X = \{x_1, \dots, x_n\}$ be a finite set, and let A be a SIVHFMS on X . Then, a single and interval-valued hybrid enthalpy fuzzy set (SIVHEFS) is given with

$$E_A = \{\langle x_i, (M_A(x_i), [M_{AL}(x_i), M_{AU}(x_i)]), (C_A(x_i), [C_{AL}(x_i), C_{AU}(x_i)]) \rangle : i = 1, \dots, n\}, \quad (8)$$

where

$$\begin{aligned} M_A(x_i) &= \frac{1}{\delta_i} SM_A, \\ M_{AL}(x_i) &= \frac{1}{\tau_i} SM_{AL}, \\ M_{AU}(x_i) &= \frac{1}{\tau_i} SM_{AU}, \end{aligned} \quad (9)$$

$$\begin{aligned} C_A(x_i) &:= C\left(\left\{\mu_A^j(x_i)\right\}_{j=1}^{\delta_i}\right), \\ C_{AL}(x_i) &:= C\left(\left\{\mu_{AL}^j(x_i)\right\}_{j=\delta_i+1}^{\delta_i+\tau_i}\right) \wedge C\left(\left\{\mu_{AU}^j(x_i)\right\}_{j=\delta_i+1}^{\delta_i+\tau_i}\right), \\ C_{AU}(x_i) &:= C\left(\left\{\mu_{AL}^j(x_i)\right\}_{j=\delta_i+1}^{\delta_i+\tau_i}\right) \vee C\left(\left\{\mu_{AU}^j(x_i)\right\}_{j=\delta_i+1}^{\delta_i+\tau_i}\right). \end{aligned} \quad (10)$$

Now, we construct a similarity measure based on Choquet integral between SIVHEFSs. For this aim, we recall concepts of fuzzy measure and Choquet integral.

Definition 4. Let X be a finite set, and let $P(X)$ be the power set of X . If

- (i) $\sigma(\emptyset) = 0$,
- (ii) $\sigma(X) = 1$,
- (iii) $\sigma(A) \leq \sigma(B)$ for any $A, B \subset X$ such that $A \subset B$ (monotonicity), then the set function $\sigma: P(X) \longrightarrow [0, 1]$ is called a fuzzy measure on X .

Definition 5. Let $X \neq \emptyset$ be a finite set, and let σ be a fuzzy measure on X . The Choquet integral of a function $f: X \longrightarrow [0, 1]$ with respect to σ is defined by

$$(C) \int_X f d\sigma := \sum_{k=1}^n (f(x_{(k)}) - f(x_{(k-1)})) \sigma(E_{(k)}), \quad (11)$$

where the sequence $\{x_{(k)}\}_{k=0}^n$ is a permutation of the sequence $\{x_k\}_{k=0}^n$ such that $0 := f(x_{(0)}) \leq f(x_{(1)}) \leq f(x_{(2)}) \leq \dots \leq f(x_{(n)})$ and $E_{(k)} := \{x_{(k)}, x_{(k+1)}, \dots, x_{(n)}\}$.

Definition 6. Let $X = \{x_1, \dots, x_n\}$ be a finite set, and let E_A and E_B be two SIVHEFSs in X , and let σ be a fuzzy measure on X . A Choquet similarity measure between A and B is given with

$$S_\sigma(E_A, E_B) := (C) \int_X f_{A,B}(x_i) d\sigma(x_i), \quad (12)$$

where

$$\begin{aligned} f_{A,B}(x_i) &:= \frac{1}{4} \left\{ \frac{C_A(x_i)C_B(x_i) + C_{AL}(x_i)C_{BL}(x_i)}{\sqrt{C_A^2(x_i) + C_{AL}^2(x_i)} \sqrt{C_B^2(x_i) + C_{BL}^2(x_i)}} \right. \\ &\quad + \frac{C_A(x_i)C_B(x_i) + C_{AU}(x_i)C_{BU}(x_i)}{\sqrt{C_A^2(x_i) + C_{AU}^2(x_i)} \sqrt{C_B^2(x_i) + C_{BU}^2(x_i)}} \\ &\quad + \frac{M_A(x_i)M_B(x_i) + M_{AL}(x_i)M_{BL}(x_i)}{\sqrt{M_A^2(x_i) + M_{AL}^2(x_i)} \sqrt{M_B^2(x_i) + M_{BL}^2(x_i)}} \\ &\quad \left. + \frac{M_A(x_i)M_B(x_i) + M_{AU}(x_i)M_{BU}(x_i)}{\sqrt{M_A^2(x_i) + M_{AU}^2(x_i)} \sqrt{M_B^2(x_i) + M_{BU}^2(x_i)}} \right\}, \end{aligned} \quad (13)$$

for $i = 1, \dots, n$.

Proposition 1. Function S_σ satisfies the following properties:

- (P₁) $0 \leq S_\sigma(E_A, E_B) \leq 1$
- (P₂) $S_\sigma(E_A, E_B) = S_\sigma(E_B, E_A)$
- (P₃) If $E_A = E_B$ then $S_\sigma(E_A, E_B) = 1$

Proof

(P₁): since $f_{A,B}(x_i)$ is the arithmetic mean of four cosine values, we have $f_{A,B}(x_i) \in [0, 1]$ for any $i = 1, \dots, n$ and the Choquet integral is monotone, we get $0 \leq S_\sigma(E_A, E_B) \leq 1$.

(P₂): it is trivial that since $f_{A,B}(x_i) = f_{B,A}(x_i)$ for any $i = 1, \dots, n$.

(P₃): if $E_A = E_B$, then we have $C_A = C_B, C_{AL} = C_{BL}, C_{AU} = C_{BU}, M_A = M_B, M_{AL} = M_{BL}$, and $M_{AU} = M_{BU}$. Therefore, we get $f_{A,B}(x_i) = 1$ for any $i = 1, \dots, n$. Since Choquet integral is an aggregation operator, we have $S_\sigma(E_A, E_B) = 1$. Thus, the proof is completed. $\square$

3. Enthalpy-TOPSIS Method for MCGDM

In this section, we provide a TOPSIS method for MCGDM problems. The TOPSIS technique is based on the shortest distance to an ideal solution to define ideal positive and negative solutions. In a sense, a positive ideal solution is a combination of the best possible criteria values, while an

ideal negative solution is a combination of the worst possible criteria values. This technique allows for the inclusion of several types of variables in the model based on their positive or negative impact on the decision-making aim, as well as the weights and degrees of importance of each criterion. The examination takes into account both quantitative and qualitative criteria, and a large number of criteria and possibilities are assessed. This technique is simple and easy to implement. Moreover, the TOPSIS method has been compared to other MCDM methods. For example, Zlaugotne et al. [28] have presented a study that compares ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), TOPSIS, multiobjective optimization by a ratio analysis plus the full multiplicative form (MULTIMOORA), preference ranking organization method for enrichment of evaluation (PROMETHEE), and complex proportional assessment (COPRAS) methods. As a result, they obtained that it is not really objective to compare the results obtained by different methods because results are similar but not the same. Since the TOPSIS method is a distance-based method, it is usually compared with VIKOR, which is also a distance-based method. For instance, Ceballos et al. [29] presented a comparative analysis for TOPSIS, VIKOR, and MULTIMOORA, and they reached that VIKOR's ranking is very sensitive to the parameter which is the coefficient of the decision mechanism. Dey et al. [30] and Lin et al. [31] presented a comparison of TOPSIS and VIKOR methods in various ranking problems. According to Opricovic and Tzeng's [32] comparative analysis of VIKOR and TOPSIS, the VIKOR method and TOPSIS method use different aggregation functions and normalization methods. The TOPSIS method is based on the principle that the optimal point should be furthest away from the positive ideal solution and the closest to the negative ideal solution. As a result, this method is appropriate for cautious (risk avoider) decision makers because the decision maker may want to make a decision that not only maximizes profit but also minimizes risk [33]. We apply this method to a real-life MCDM problem. Before, we give a score algorithm for SIVHEFVs. Let

$$\alpha = \langle (M_\alpha, [M_{\alpha L}, M_{\alpha U}]), (C_\alpha, [C_{\alpha L}, C_{\alpha U}]) \rangle. \quad (14)$$

Then, a score function is given with

$$S(\alpha) = \frac{2M_\alpha + M_{\alpha L} + M_{\alpha U} + 2C_\alpha + C_{\alpha L} + C_{\alpha U}}{8}. \quad (15)$$

(1) If $S(\alpha) < S(\beta)$, then we say $\alpha < \beta$

(2) If $S(\alpha) = S(\beta)$ and

- (I) $[M_{\alpha L}, M_{\alpha U}] <_{A,B} [M_{\beta L}, M_{\beta U}]$, then $\alpha < \beta$
- (II) $[M_{\alpha L}, M_{\alpha U}] = [M_{\beta L}, M_{\beta U}]$ and
 - (a) $M_\alpha < M_\beta$, then $\alpha < \beta$
 - (b) $M_\alpha = M_\beta$ and
 - (i) $[C_{\alpha L}, C_{\alpha U}] <_{A,B} [C_{\beta L}, C_{\beta U}]$, then $\alpha < \beta$
 - (ii) $[C_{\alpha L}, C_{\alpha U}] = [C_{\beta L}, C_{\beta U}]$ and $C_\alpha < C_\beta$, then $\alpha < \beta$

where $<_{A,B}$ is the linear order given in Lemma 1 of [34] by $[x, y] <_{A,B} [u, v]$ whenever " $A(x, y) < A(u, v)$ or $A(x, y) =$

$A(u, v)$ and $B(x, y) \leq B(u, v)$ and $A(x, y) = A(u, v)$ " and $A, B: [0, 1]^2 \rightarrow [0, 1]$ are aggregation functions (see [35]).

3.1. Steps of the Enthalpy-TOPSIS Method. We consider a MCGDM problem with the set of alternatives $\{A_1, \dots, A_m\}$ and the set of criteria $\{x_1, \dots, x_n\}$.

Step 1: δ number of DMs evaluate the alternatives by using fuzzy values, and τ number of DMs evaluate the alternatives by using interval-valued fuzzy values. So, each alternative A_k is presented as a SIVHFMS with equal sequence lengths $\delta + \tau$ for any $k = 1, \dots, m$.

Step 2: each A_k is converted to SIVHEFS E_{A_k} by using Definition 3 to aggregate DM evaluations. Thus, the decision matrix which is independent from DMs is obtained as follows:

$$E = \begin{bmatrix} E_{A_1} \\ \vdots \\ E_{A_m} \end{bmatrix} = \begin{bmatrix} \alpha_1^{(1)} & \dots & \alpha_n^{(1)} \\ \vdots & \ddots & \vdots \\ \alpha_1^{(m)} & \dots & \alpha_n^{(m)} \end{bmatrix}_{m \times n}, \quad (16)$$

where

$$\alpha_i^{(k)} = \left\langle x_i, \left(M_{A_k}(x_i), [M_{A_k L}(x_i), M_{A_k U}(x_i)] \right), \left(C_{A_k}(x_i), [C_{A_k L}(x_i), C_{A_k U}(x_i)] \right) \right\rangle. \quad (17)$$

Step 3: the ideal best solution $E_b = \{\alpha_i^{(b)} : i = 1, \dots, n\}$ and the ideal worst solution $E_w = \{\alpha_i^{(w)} : i = 1, \dots, n\}$ are calculated. Considering the definition of the concept of enthalpy and using the score algorithm, we see that

$$\alpha_i^{(b)} = \max_{k=1, \dots, m} \alpha_i^{(k)}. \quad (18)$$

$$\alpha_i^{(w)} = \min_{k=1, \dots, m} \alpha_i^{(k)}, \quad (19)$$

whenever x_i is a benefit criterion and

$$\alpha_i^{(b)} = \min_{k=1, \dots, m} \alpha_i^{(k)}, \quad (20)$$

$$\alpha_i^{(w)} = \max_{k=1, \dots, m} \alpha_i^{(k)}, \quad (21)$$

whenever x_i is a cost criterion.

Step 4: a fuzzy measure σ is identified over the set of criteria via a fuzzy measure identification method.

Step 5: for each $k = 1, \dots, m$, we calculate the distance between E_{A_k} and the ideal best solution

$$d_k^{(b)} = 1 - S_\sigma(E_{A_k}, E_b) \quad (22)$$

and the distance between E_{A_k} and the ideal worst solution

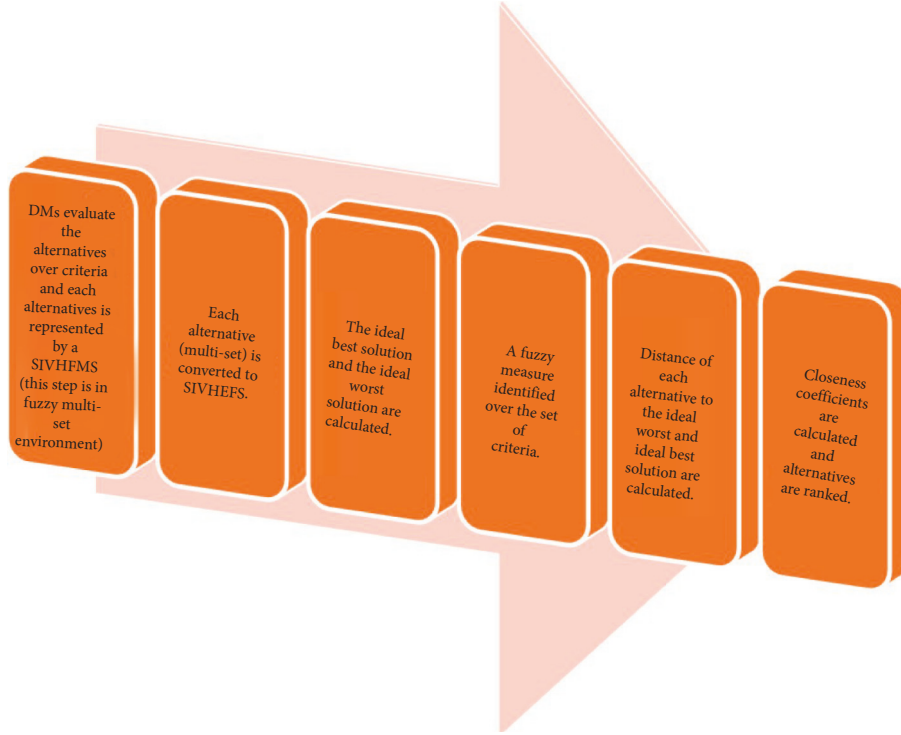


FIGURE 1: Flowchart of the Enthalpy-TOPSIS method.

$$d_k^{(w)} = 1 - S_\sigma(E_{A_k}, E_w), \quad (23)$$

where S_σ is the Choquet similarity measure with respect to σ given in Definition 6. It is noted here that using a fuzzy measure and Choquet integral instead of criteria weights and weighted arithmetic mean provides us more sensitive similarities.

Step 6: the closeness coefficients

$$s_k = \frac{d_k^{(w)}}{d_k^{(w)} + d_k^{(b)}}, \quad (24)$$

is calculated, and the alternatives are ranked. The alternative that has larger s_k value is better.

The steps of the enthalpy-TOPSIS method is illustrated in Figure 1.

3.2. A Research Assistant Selection Problem. We deal with a research assistant selection problem in mathematics department.

Step 1: three criteria are required for the selection:

$$X = \left\{ \begin{array}{l} \text{oral presentation in a foreign language } (x_1), \\ \text{oral presentation in mathematics } (x_2), \\ \text{academic background } (x_3) \end{array} \right\}. \quad (25)$$

Four graduate students A_1, A_2, A_3 , and A_4 are considered as alternatives, and four DMs evaluate the

alternatives over the criteria. Two (δ) of them are asked to make the evaluation in the fuzzy environment, and two (τ) of them are asked to make the evaluation in interval-valued fuzzy environment. Thus, the decision matrix consists of SIVHFMSs with $\delta = 2$ and $\tau = 2$. The DMs' preference values are summarized in the form of decision matrix in Table 2.

Step 2: each A_k is converted to SIVHEFS. Therefore, the decision matrix does not depend on the DMs (see Table 3)

Step 3: ideal best and ideal worst solutions are given in Table 4

Step 4: the λ -fuzzy measure σ given in Table 5 is identified over the set of criteria [27]. The weights $\omega_1 = 0.122021$, $\omega_2 = 0.229641$, and $\omega_3 = 0.648337$.

Step 5: for each $k = 1, \dots, 4$, we calculate $d_k^{(b)}$ and $d_k^{(w)}$ (see Table 6)

Step 6: the closeness coefficient s_k is obtained as follows: $s_1 = 0.3516$, $s_2 = 0.47785$, $s_3 = 0.60852$, and $s_4 = 0.58048$ for each $k = 1, \dots, 4$. Hence, we have $A_3 > A_4 > A_2 > A_1$.

3.3. Comparison Analysis. In this subsection, we compare the result of the present study with the result of the TOPSIS method. For this purpose, we aggregate the values given in Table 2 by using arithmetic mean and we get the decision matrix given in Table 7. For the intervals, we consider the midpoints.

Using the same weights $\omega_1 = 0.122021$, $\omega_2 = 0.229641$, and $\omega_3 = 0.648337$ in the TOPSIS method, we get the

TABLE 2: Decision matrix.

A_1	$\langle x_1, \{0.7, 0.6, [0.4, 0.5], [0.6, 0.8]\} \rangle$
	$\langle x_2, \{0.3, 0.6, [0.4, 0.7], [0.6, 0.7]\} \rangle$
	$\langle x_3, \{0.4, 0.5, [0.3, 0.5], [0.4, 0.6]\} \rangle$
A_2	$\langle x_1, \{0.8, 0.7, [0.6, 0.7], [0.6, 0.8]\} \rangle$
	$\langle x_2, \{0.5, 0.4, [0.5, 0.6], [0.4, 0.5]\} \rangle$
	$\langle x_3, \{0.6, 0.4, [0.5, 0.7], [0.3, 0.4]\} \rangle$
A_3	$\langle x_1, \{0.5, 0.3, [0.4, 0.6], [0.5, 0.6]\} \rangle$
	$\langle x_2, \{0.8, 0.9, [0.7, 0.9], [0.8, 0.9]\} \rangle$
	$\langle x_3, \{0.7, 0.6, [0.5, 0.6], [0.6, 0.7]\} \rangle$
A_4	$\langle x_1, \{0.6, 0.7, [0.5, 0.6], [0.6, 0.8]\} \rangle$
	$\langle x_2, \{0.6, 0.7, [0.5, 0.6], [0.4, 0.5]\} \rangle$
	$\langle x_3, \{0.5, 0.4, [0.3, 0.5], [0.4, 0.5]\} \rangle$

TABLE 3: Decision matrix independent from DMs.

A_1	$\langle x_1, (0.65, [0.5, 0.65]), (0.6549, [0.6635, 0.6669]) \rangle$
	$\langle x_2, (0.45, [0.5, 0.7]), (0.6817, [0.6534, 0.6635]) \rangle$
	$\langle x_3, (0.45, [0.35, 0.55]), (0.6565, [0.6555, 0.6585]) \rangle$
A_2	$\langle x_1, (0.75, [0.6, 0.75]), (0.6545, [0.6534, 0.6545]) \rangle$
	$\langle x_2, (0.45, [0.45, 0.55]), (0.6565, [0.6555, 0.6565]) \rangle$
	$\langle x_3, (0.5, [0.4, 0.55]), (0.6635, [0.6692, 0.6723]) \rangle$
A_3	$\langle x_1, (0.4, [0.45, 0.6]), (0.6692, [0.6534, 0.6565]) \rangle$
	$\langle x_2, (0.85, [0.75, 0.9]), (0.6543, [0.6534, 0.6545]) \rangle$
	$\langle x_3, (0.65, [0.55, 0.65]), (0.6549, [0.6549, 0.6555]) \rangle$
A_4	$\langle x_1, (0.65, [0.55, 0.7]), (0.6549, [0.6555, 0.6585]) \rangle$
	$\langle x_2, (0.65, [0.45, 0.55]), (0.6549, [0.6555, 0.6565]) \rangle$
	$\langle x_3, (0.45, [0.35, 0.5]), (0.6565, [0.6534, 0.6585]) \rangle$

TABLE 4: Ideal best and ideal worst solutions.

E_b	$\langle x_1, (0.75, [0.6, 0.75]), (0.6545, [0.6534, 0.6545]) \rangle$
	$\langle x_2, (0.85, [0.75, 0.9]), (0.6543, [0.6534, 0.6545]) \rangle$
	$\langle x_3, (0.65, [0.55, 0.65]), (0.6549, [0.6549, 0.6555]) \rangle$
E_w	$\langle x_1, (0.4, [0.45, 0.6]), (0.6692, [0.6534, 0.6565]) \rangle$
	$\langle x_2, (0.45, [0.45, 0.55]), (0.6565, [0.6555, 0.6565]) \rangle$
	$\langle x_3, (0.45, [0.35, 0.5]), (0.6565, [0.6534, 0.6585]) \rangle$

TABLE 5: Fuzzy measure σ .

$\sigma(\emptyset) = 0$	$\sigma(x_1) = 0.280218$
$\sigma(x_2) = 0.467382$	$\sigma(x_3) = 0.868646$
$\sigma(x_1, x_2) = 0.628345$	$\sigma(x_1, x_3) = 0.927225$
$\sigma(x_2, x_3) = 0.966351$	$\sigma(x_1, x_2, x_3) = 1$

TABLE 6: Distances to ideal best and ideal worst solutions.

$d_1^{(b)} = 0.001396$	$d_2^{(b)} = 0.00033$
$d_3^{(b)} = 0.000285$	$d_4^{(b)} = 0.000675$
$d_1^{(w)} = 0.000757$	$d_2^{(w)} = 0.000302$
$d_3^{(w)} = 0.00443$	$d_4^{(w)} = 0.000934$

closeness coefficients $s_1 = 0.1323$, $s_2 = 0.2680$, $s_3 = 0.8470$, and $s_4 = 0.1637$. So, we have $A_3 > A_2 > A_4 > A_1$. It is noted that the best and the worst chooses are same.

TABLE 7: Aggregated decision matrix.

	x_1	x_2	x_3
A_1	0.6125	0.5250	0.4500
A_2	0.7125	0.4750	0.4875
A_3	0.4625	0.8375	0.6250
A_4	0.6375	0.5750	0.4375

4. Conclusion

In this paper, we define the notion of SIVHEFS. In a FMS and IVFMS environment, a SIVHEFS based on average and enthalpy values can offer useful hybrid information about sequences. SIVHEFS is a concept that decreases the reliance of information in FMS and IFMS on sequence length and provides hybrid information in a more compact manner. Moreover, it contains statistical data such as the average of membership sequences and an enthalpy value calculated using Shannon's entropy. We provide a cosine similarity measure between SIVHEFSs based on the Choquet integral. Since Choquet integral considers the interaction among criteria, the proposed similarity measure provides a sensitive similarity analysis in the fuzzy multiset environment in contrast to the weighted average. Later, we give a TOPSIS approach for MCGDM problems. The complement of the proposed cosine similarity measure is used to measure the distance between alternatives in this TOPSIS approach. We apply the proposed approach to a real life problem. We also compare the results of the paper with a classical TOPSIS method. The created enthalpy-TOPSIS approach not only increases decision-making dependability but also provides DMs with a new impact pathway. In the future, hybrid MCDM methods, which are quite up-to-date [36,37] will be discussed in the proposed new fuzzy environment.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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