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ABSTRACT

In this study, we carried out the analysis of the resolution time of Josephson junction (JJ) balanced comparators with anharmonic current-phase relation (CPR). An expression for the time resolution was obtained in the case of linearly growing strobe current pulse driven via the junction. It is shown that the resolution time of balanced comparators is improved by the renormalized critical current of JJ with anharmonic CPR.

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1. INTRODUCTION

The high nonlinearity of the current-voltage characteristic of Josephson junctions (JJs) makes them promising for superconducting electronics.[1,2] Super-high-speed switches have been built with complex distributed JJs and interferometers, where switching is induced by suppressing the critical current of the system.[3] Switching cells on JJs can be viewed as building blocks for different superconducting devices. Josephson comparators are the main structural elements of modern superconducting electronics.[4–6] First Josephson comparators were based on two tunnel junctions with different critical currents. One of the junctions acted as a generator of short strobe pulses, while the other acted as a comparator.[7] However, thermal and technological fluctuations in the comparator circuit with a single JJ have a bad impact on the characteristics of the comparator and make them sensitive to various fluctuations. To exclude the influence of various fluctuations, a balanced circuit of JJs was proposed based on the Goto pair principle.[8,9] Goto pair is the case when two identical junctions are connected in series with the strobe pulse while in parallel with the measurement signal. This method was proposed many years ago and used in the construction of semiconductor circuits based on Esaki diodes.[10–12] It should be noted that the use of the balanced principle makes a comparator insensitive to the noise and drift of the strobe generator. The sensitivity of Josephson comparators is defined by thermal or quantum fluctuations, and the time resolution is determined by the inertial properties of the junctions. Josephson comparators can also be used in the construction of superconducting samplers,[13] which are used to measure weak and ultrashort pulses.[13,14] The dynamics of the processes and the user characteristics of samplers were numerically simulated using a PSCAN program package.[15,16]

In all aforementioned studies of the dynamical properties of JJ based circuits (see Refs. 1 and 4, it was considered that current-phase relation (CPR) has a harmonic character with critical current I_c

$$I = I_c \sin\varphi. \tag{1}$$

Relationship (1) is fulfilled with high accuracy for tunnel JJs based on low-temperature superconductors.[17] In the case of JJs based on high-temperature superconductors, the CPR becomes anharmonic,[18,19]

$$I = I_c(\alpha) f(\varphi) = I_c(\alpha)(\sin\varphi + \alpha \sin 2\varphi), \tag{2}$$

where anharmonicity parameter α depends on the junction preparation technology. In general, anharmonicity in the CPR for high temperature and Fe-based superconductors-based JJ for JJs made of high-temperature and Fe-based superconductors is associated with the *d* wave behavior of the order parameter and multiband character of the superconducting state in new superconducting

compounds.[20] The dynamical properties of single JJs with an anharmonic CPR (2) were studied in Refs. 21–23. The influence of the second harmonic in the CPR of a JJ on the time resolution of balanced comparators was not considered yet. In this study we investigate the influence of the anharmonicity of CPR on the time resolution of the Josephson balanced comparator.

2. BASIC EQUATIONS

For the analysis of the dynamics of JJs with anharmonic CPR (2), we use a differential equation for the dynamics of the Josephson phase φ

$$\beta\ddot{\varphi} + \dot{\varphi} + f(\varphi) = i_e, \tag{3}$$

where i_e is the current via JJ, dots over φ correspond to derivative with respect to the dimensionless time $\frac{\Phi_0}{2\pi I_c(\alpha)R_N}$. In Eq. (3) notation β is the McCumber parameter of JJ $\beta = \frac{2e}{\hbar} I_c(\alpha) R_N^2 C$, which determines the size of hysteresis in the current-voltage characteristics. It is well known, that the case of β >> 1 corresponds to the tunnel JJ. In the case of β << 1 the hysteresis in current-voltage characteristics is absent and in Eq. (3) we can neglect the first term. The presence of the second harmonic in CPR in Eq. (2) leads to renormalization of the critical current I_c. In calculations, we use an analytical solution for the maximum point of the function f (φ) (2), similar to Refs. 24 and 25.

$$F(\varphi) = (\sin\varphi + \alpha \sin 2\varphi). \tag{4}$$

The effective critical current $I_c(\alpha)$ of a JJ with an anharmonic current-phase relation as a function of anharmonicity parameter a is presented in Refs. 24 and 25. It is clear that, by increasing parameter α, the effective critical current I_c also increases. Quadratic behavior at small α changes for linear dependence at high values of this parameter α.

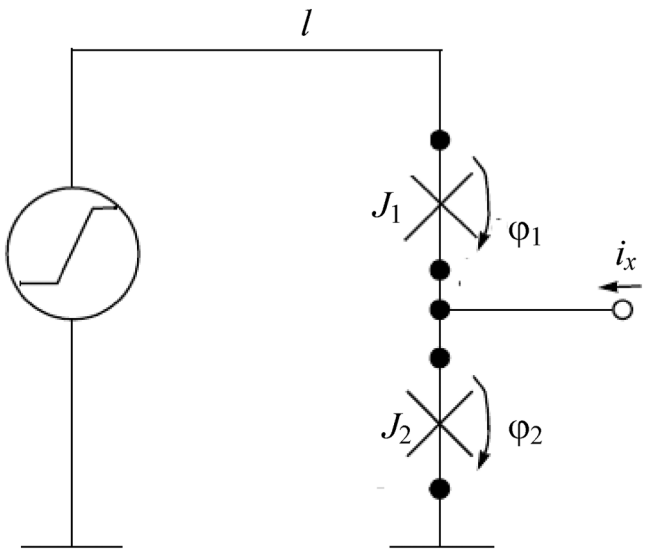


FIG. 1. Equivalent scheme of a balanced comparator on JJs.

Equivalent circuit of the balanced comparator based on JJs in general case is presented in Fig. 1. It is clear that, two identical Josephson junctions (J_1 and J_2) are connected in series with the strobe pulse i and in parallel with the measurement signal i_x. Strobe pulses in Fig. 1 generate a linear growing current. As for any comparator, the value of the time resolution can be determined using the transient response $H(\tau)$,[4] which represents the output signal of the comparator V_{out}, when the signal in the form of a small current step $I_S = I\theta(t)$ is applied to the input of the comparator. In this case, the time resolution is usually defined as the step response time from the level of 0.10 to 0.90 of its maximum value, where τ denotes the delay time (lead time) of the strobe pulse relative to the step. For the anharmonic CPR Josephson comparator with junctions J_1 and J_2 (Fig. 1), the following equations hold:[4,13]

$$\begin{aligned} \beta\ddot{\varphi}_1 + \dot{\varphi}_1 + f(\varphi_1) &= i, \\ \beta\ddot{\varphi}_2 + \dot{\varphi}_2 + f(\varphi_2) &= i + i_x. \end{aligned} \tag{5}$$

The dynamic behavior is determined by two control variables i and i_x. The strobe pulse equally affects both phases of both JJs, while i_x causes opposite changes of the phase of the junctions. For small phase increments, $\delta\varphi = \varphi_2 - \varphi_1$, we have the following equations:

$$\begin{aligned} \beta\delta\ddot{\varphi}_1 + \delta\dot{\varphi}_1 + f(\varphi_2) - f(\varphi_1) &= i_x, \\ \beta\ddot{\varphi}_+ + \dot{\varphi}_+ + f(\varphi_2) + f(\varphi_1) &= 2i + i_x, \end{aligned} \tag{6}$$

where φ_+ is $\varphi_+ = \varphi_1 + \varphi_2$. Under the condition of the small measured signal in relation to the sampling signal $i_x << i$

$$\begin{aligned} \beta\delta\ddot{\varphi} + \delta\dot{\varphi} + \frac{\partial f}{\partial\varphi}(\varphi_1)\delta\varphi &= i_x, \\ \beta\frac{\ddot{\varphi}_+}{2} + \frac{\dot{\varphi}_+}{2} + \frac{f(\varphi_2) + f(\varphi_1)}{2} &= i, \end{aligned} \tag{7}$$

the system of Eq. (7) can be rewritten as

$$\begin{aligned} \beta\delta\ddot{\varphi} + \delta\dot{\varphi} + \frac{\partial f}{\partial\varphi}\left(\frac{\varphi_+}{2}\right)\delta\varphi &= i_x, \\ \beta\frac{\ddot{\varphi}_+}{2} + \frac{\dot{\varphi}_+}{2} + f\left(\frac{\varphi_+}{2}\right) &= i. \end{aligned} \tag{8}$$

An asymptotic solution of the second Eq. (8) with harmonic CPR (α = 0) in the case of linear sampling current $i = rt$ is presented in Ref. 26 (see also Ref. 1). Here r denotes the dimensionless growing rate of the current via JJ

$$r = \frac{d(I/I_c)}{d(\omega_c t)}, \tag{9}$$

where $\omega_c = \frac{2eI_cR_N}{\hbar}$ is the characteristic frequency of JJ.

Balanced comparator using Single Flux Quantum (SFQ) technique is discussed in Ref. 9 (see also Ref. 4. It is well known that SFQ technology uses overdamped JJ.[4] It means that, in this limit,

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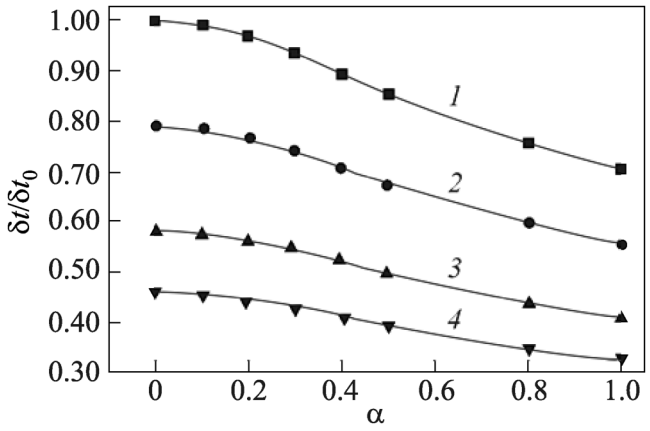


FIG. 2. Time resolution of tunnel JJ balanced comparators as a function of a nharmonicity for different sampling rates $r = 1, 2, 5, 10$ from top to bottom.

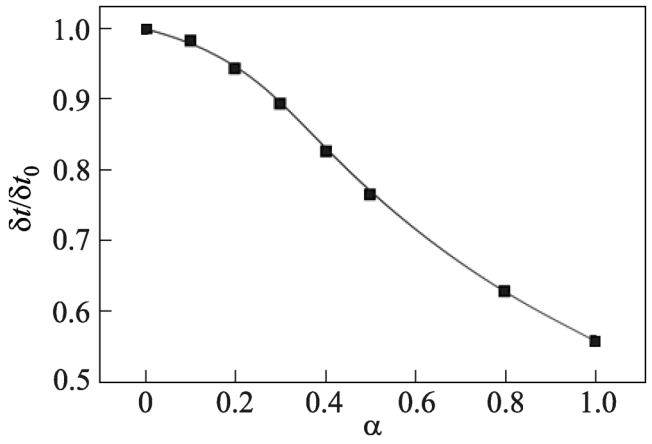


FIG. 3. Time resolution of overdamped JJ balanced comparators as a function of anharmonicity for SFQ pulses.

the first term is ignored in Eqs. (5) and (6) due to $\beta << 1$

$$\delta\dot{\varphi} + \frac{\partial f}{\partial \varphi}\left(\frac{\varphi_+}{2}\right)\delta\varphi = i_x,$$
$$\frac{\dot{\varphi}_+}{2} + f\left(\frac{\varphi_+}{2}\right) = i. \tag{10}$$

3. RESULTS

Calculation for the time resolution of tunnel JJ-based balanced comparators using Eq. (8), considering anharmonicity effects lead to a result

$$\delta t = 2.3 r^{-1/3}\beta^{2/5}(\alpha)\frac{\Phi_0}{2\pi I_c(\alpha)R_N}. \tag{11}$$

In expression (11) we consider renormalization of the McCumber parameter $\beta(\alpha)$ and the critical current $I_c(\alpha)$ due to anharmonicity of CPR.[24,25] Similar calculation for the case of overdamped JJ for SFQ pulses has a form

$$\delta t = 0.64\frac{\Phi_0}{2\pi I_c(\alpha)R_N}. \tag{12}$$

In the derivation of (12), we consider the triangle forms of the strobe pulses while the dimensionless sampling rate is fixed to $r = 2/\pi$ (SFQ pulse area is equal to the flux quantum Φ_0).

The results of calculations of the time resolution of a balanced JJ comparator are presented in Figs. 2 and 3, respectively, for the tunnel and overdamped junctions. As seen in these figures, in both cases, anharmonicity of CPR leads to improvement of the time resolution in comparison to the harmonic case $\alpha = 0$. Such a result can be explained by the decreasing of the Josephson inductance of the junction and, as a result, decreasing of its inertial properties. It is well known that the supercurrent in the Josephson junction $I = I_c(\alpha)\sin\varphi$ is equivalent to alternating differential Josephson inductance[1] $L(\varphi) = \frac{\Phi_0}{2\pi I_c(\alpha)\cos(\varphi)}$. As follows from Figs. 2 and 3, the influence of anharmonic effects of CPR in the case of overdamped JJ is higher than that of tunnel junctions. The influence of the sampling rate r in tunnel JJ comparators on the time resolution is also presented in Fig. 2 for different values of $r = 1, 2, 5, 10$ from top to bottom.

The anharmonicity parameter α is important in finding applications for Josephson junctions with an anharmonic current-phase relationship. In Ref. 27, bicrystalline Josephson junctions were fabricated in YBaCuO high-temperature superconductors, and a second to first harmonic amplitude ratio as a function of temperature was presented. At temperatures lower than 0.2 K, the influence of the second harmonic on the CPR becomes important, as follows from this dependence. This obtained result seems interesting from the point of view of wide application of balanced Josephson comparators in digital superconducting electronics[4] and flux qubit readout systems.[28]

Thus, in this paper, the resolution time of JJ balanced comparators with anharmonic CPR was investigated. Renormalization of the critical current in JJs with anharmonic CPR leads to an improvement of the resolution time with an increasing anharmonicity parameter α. The estimation shows that it is possible to reach an improvement of the time resolution of about two times.

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REFERENCES

[1]K. K. Likharev, *Introduction Into Dynamics of Josephson Junctions and Circuits* (Gordon Breach, New York, 1986), p. 586.
[2]K. K. Likharev, A. B. Zorin, and V. K. Semenov, New possibilities of superconducting electronics, in Superconducting Devices, edited by S. T. Ruggerio and D. A. Rudman (Academic Press, Boston, 1990), pp. 1–49.
[3]R. Gross and A. Marx, *Applied superconductivity: Josephson Effect and Superconducting Electronics* (Walter-Meissner-Institute, Garching, 2005), p. 215.

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[4]I. Askerzade, A. Bozbey, and M. Canturk, *Modern Aspects of Josephson Dynamics and Superconductivity Electronics* (Springer, Berlin, 2017), p. 186.
[5]K. K. Likharev, "Superconducting digital electronics," Physica C **482**, 6 (2012).
[6]T. J. Walls, T. V. Filippov, and K. K. Likharev, Phys. Rev. Lett. **89**, 217004 (2002).
[7]S. M. Faris, Appl. Phys. Lett. **36**, 1005 (1980).
[8]P. Bakhtin, P. E. Kandyba, V. I. Makhova, and G. Y. Pavlov, *Josephson Balanced Comparators*, USSR Inventors Certificate No. 1008908 (1983).
[9]V. K. Kornev and V. K. Semenov, Proc. of Int. Supercond. Electr. Conf. (ISEC'87), Tokyo (1987), p. 131.
[10]V. G. Karklinsh and E. Khermanis, *Two Site Frequency Transformers* (Zinatne, Riga, 1980), p. 321.
[11]E. Beiner and K. Kruminsh, Aut. Cont. Compt. Sci. **45**, 314 (2011).
[12]K. Kruminsh and V. Plotsinsh, Aut. Cont. Compt. Sci. **47**, 283 (2013).
[13]I. N. Askerzade, Tech. Phys. **51**, 393 (2006).
[14]M. Hidaka, M. Maruyama, and S. Tahara, Physica C **378**, 1435 (2002).
[15]I. N. Askerzade and V. K. Kornev, Radiotekh. Elektron. **39**, 869 (1994).
[16]I. N. Askerzade, Tech. Phys. **45**, 66 (2000).
[17]E. İlichev, V. Zakosarenko, L. Fritzsch, and R. Stolz, Rev. Sci. Instr. **72**, 1882 (2001).
[18]T. Bauch, F. Lombardi, F. Tafuri, A. Barone, G. Rotoli, P. Delsing, and T. Claeson, Phys. Rev. Lett. **94**, 087003 (2005).
[19]M. N. S. Amin, A. Y. Smirnov, A. M. Zagoskin, T. Lindström, S. A. Charlebois, T. Claeson, and A. Y. Tzalenchuk, Phys. Rev. B **73**, 064516 (2005).
[20]I. Askerzade, *Unconventional Superconductors: Anisotropy and Multiband Effects* (Springer, Berlin, 2012), p. 177.
[21]I. N. Askerzade, Fiz. Nizk. Temp. **41**, 315 (2015) [Low Temp. Phys. **41**, 241 (2015)].
[22]M. Canturk and I. N. Askerzade, IEEE Trans. Appl. Supercond. **21**, 3541 (2011).
[23]M. Canturk and I. N. Askerzade, IEEE Trans. Appl. Supercond. **22**, 1400106 (2012).
[24]E. Goldobin, D. Koelle, R. Kleiner, and A. Buzdin, Phys. Rev. B **76**, 224523 (2007).
[25]I. N. Askerzade and R. Askerbeyli, Fiz. Nizk. Temp. **46**, 1088 (2020) [Low Temp. Phys. **46**, 919 (2020)].
[26]O. V. Snigirev, Comm. Tech. and Elect. **29**, 2216 (1994).
[27]G. Ovsyannikov and K. I. Konstantinyan, Fiz. Nizk. Temp. **38**, 423 (2012) [Low Temp. Phys. **38**, 333 (2012)].
[28]T. J. Walls, D. V. Averin, and K. Likharev, IEEE Trans. Appl. Supercond. **17**, 136 (2007).

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